present environments. The calculation procedure is that described in Ref. 20.

In Fig. 3, the ablation rates $\dot{m}$ obtained by the present method are shown for typical conditions and are compared with those obtained using Eq. (1). As seen in the figure, the present results agree closely with those of Eq. (1) for $R=10$ cm, except for a narrow temperature range $1000<T_w<1400$ K where the difference in ϵ_l values between Eqs. (1) and (2) influences the results. For the low-density, large-R conditions, the present calculations clearly predict greater ablation rates. The increase is because of oxidation by atomic oxygen.

The integrated mass loss $\int \dot{m} dt$ was calculated for a group of flight trajectories. The trajectories considered start from the geosynchronous orbit and enter the Earth's atmosphere many times in a skipping motion, following near-elliptic, decaying orbits. The calculations were made assuming zero lift and were terminated when the sum of kinetic and potential energies of the vehicle reached that of the space shuttle orbit. The nose radius and the ballistic coefficients were taken[22] to be $R=426.5$ cm and $M/C_dA=46.7$ g/cm^2, corresponding roughly to a truncated hemisphere-cylinder of 426.5-cm radius and 426.5-cm cylinder diameter with 10,000-kg mass and drag coefficient of $C_d=1.5$. The orbit calculation was performed under a simplifying assumption that the entries occur only in the Earth's equatorial plane from west to east. The present results are compared in the figure with those calculated by using Eq. (1). In applying Eq. (1), heat-transfer rates and corresponding wall temperatures were calculated by three different methods, i.e., the present model, Fay and Riddell's[23] high-limit value that assumes the chemical energy of dissociation to be transmitted completely into the wall, and the low-limit value that excludes the chemical energy.

As seen in Fig. 4, for the low-perigee, low number-of-pass (i.e., steep) trajectories, the present calculation and those by Eq. (1) are in excellent agreement. The agreement is expected because the flow is in the diffusion-controlled regime where the surface kinetics are immaterial. However, for high-perigee, high number-of-pass (i.e., shallow) trajectories for which surface kinetics are likely to be important, the present calculation yields mass loss values much greater than Eq. (1), i.e., up to three orders of magnitude, depending on the method used for computing the heat-transfer rate. The difference is because of the surface oxidation by atomic oxygen.

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Thermocouple Time Constant Measurement by Cross Power Spectra

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Introduction

THE measurement of fluctuating temperatures downstream of the combustor in turbopropulsion systems is required to quantify the importance of entropy noise generation in these systems.[1-3] It is well known, however, that most thermocouples suitable for use in such a hostile environment have response times considerably longer than required for flat response in the audible frequency range. Consequently, they must be compensated. Central to the compensation problem is the problem of measuring the response time of a given thermocouple, because it must be measured in the environment which it will see in use. This is so because the

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response time depends upon the convective and radiative environment in which the thermocouple is located.[4] Methods previously developed for measurement of the time constant have usually relied on measurement of the impulse response by quickly inserting the thermocouple into the environment, switching a gas stream from cold to hot, or by turning off an initial current through the wire and watching the temperature decay to the temperature of the environment.[5] Mechanical methods are adequate for time constants of the order of 100 msec or longer. However, for "fast" response thermocouples (<10 msec), the mechanical switching methods are not usually fast enough to provide a true step input temperature change. Electrical methods, since they involve heating of the wire above the environment temperature, are not satisfactory if the thermocouple is operating near its limit of survivability. The purpose of this note is to show a new method of thermocouple time constant measurement, which at the same time is nonintrusive upon the thermocouple.

Analysis

The fundamental assumptions are the equation for the AC output of the thermocouple, y, which is proportional to the apparent temperature fluctuation about the mean temperature, is given by $y=\bar{y}+z$, where $\bar{y}$ is the portion of the signal truly proportional to the temperature flucturations, and z is the background electronic noise. $\bar{y}$ obeys the linear differential equation

$$\frac{d\bar{y}}{dt}=\frac{1}{\tau}(\eta x-\bar{y}) \tag{1}$$

Here τ is the time constant, x is the true temperature fluctuation, and t is time. η is a factor, usually very near unity, which may be calculated from known radiation and conduction correction factors.[4] η is the ratio of the equilibrium thermocouple reading to the reading it would give if it were at the true temperature. The method of measurement of τ is the issue. In the method outlined here, two thermocouples, denoted by 1 and 2, of differing time constants are required to be placed in the flow as closely as possible to one another so as to see the same temperature history. Any separation of the two thermocouples will yield some error, the exact nature of which is discussed later.

The method assumes the availability of equipment which will readily process the signals by Fourier analysis. Taking the finite Fourier transform of Eq. (1),

$$\bar{Y}(1+i\omega\tau)=\eta X$$

and by definition

$$Y=\bar{Y}+Z=\frac{\eta X}{1+i\omega\tau}+Z \tag{2}$$

Here capital letters stand for the Fourier transform of the small letter quantities. The cross power spectrum and the auto power spectra are‡

$$S_{12}=Y_1Y_2^* \qquad S_{11}=Y_1Y_1^*$$

$$S_{22}=Y_2Y_2^*$$

which yield from Eq. (2)

$$S_{12}=\left(\frac{\eta X}{1+i\omega\tau_1}+Z_1\right)\left(\frac{\eta X^*}{1-i\omega\tau_2}+Z_2^*\right)$$

$$S_{11}=\left(\frac{\eta X}{1+i\omega\tau_1}+Z_1\right)\left(\frac{\eta X^*}{1-i\omega\tau_1}+Z_1^*\right) \tag{3}$$

‡Here, because they will cancel out later, proportionality constants in the spectral functions involving the sampling time have been omitted.

where a * denotes the complex conjugate. If the spurious noise is incoherent with the signal then an ensemble average, denoted by a bar superscript, will yield

$$\bar{S}_{12}=\frac{\eta^2\overline{XX^*}}{1+\omega^2\tau_1\tau_2+i\omega(\tau_1-\tau_2)}$$

$$\bar{S}_{11}=\frac{\eta^2\overline{XX^*}}{(1+\omega^2\tau_1^2)}+\overline{Z_1Z_1^*} \tag{4}$$

provided that z_1 is incoherent with z_2. In what follows it is presumed that the spurious noise for signal 1 is sufficiently small that $\overline{Z_1Z_2^*}$ may be neglected in comparison with the first term in $\bar{S}_{11}$. Now constructing the ratio of $\bar{S}_{11}$ to $\bar{S}_{12}$,

$$R\equiv\bar{S}_{11}/\bar{S}_{12}=\frac{1+\omega^2\tau_1\tau_2+i\omega(\tau_1-\tau_2)}{1+\omega^2\tau_1{}^2} \tag{5}$$

Consider then

$$\lim_{\omega\to\infty}R=R_r=\tau_2/\tau_1 \tag{6}$$

which directly gives the time constant ratio. More importantly, consider the imaginary part of R, R_i.

$$R_i=\frac{\omega(\tau_1-\tau_2)}{1+\omega^2\tau_1{}^2}$$

Differentiating, and finding an extremum in this function,

$$\frac{\tau_1-\tau_2}{1+\omega_e^2\tau_1{}^2}-\frac{2\omega_e^2(\tau_1-\tau_2)\tau_1{}^2}{(1+\omega_e^2\tau_1{}^2)^2}=0$$

where the e subscript denotes an extremum value of ω. For $\tau_1\neq\tau_2$, which is the reason for using two different thermocouples, there is a single extremum at

$$\omega_e=1/\tau_1 \tag{7}$$

Thus, τ_1 may be found directly if a plot of R_i is available. The primary restrictions here are that both thermocouples are seeing the same x and that the noise does not mask the signal in the vicinity of ω_e.

The quality of the data can be assured by constructing the coherence function.[6] This is

$$\gamma^2\equiv\frac{\bar{S}_{12}\bar{S}_{12}^*}{\bar{S}_{11}\bar{S}_{22}}=\frac{1}{1+b}$$

$$b=\frac{\overline{Z_1Z_1}^*[1+(\omega\tau_2)^2]}{\eta^2\overline{XX^*}}+\frac{\overline{Z_2Z_2}^*[1+(\omega\tau_1)^2]}{\eta^2\overline{XX^*}}$$

$$+\frac{\overline{Z_1Z_1^*Z_2Z_2^*}[\omega\tau_1)^2][1+(\omega\tau_2)^2]}{\eta^4\overline{XX^*}}$$

All the terms in b are positive, real quantities. A *sufficient* condition, therefore for Eq. (7) to be valid is one of γ^2 being close to unity in the vicinity of ω_e. This condition furthermore assures that X at position 1 is the same as X at position 2 since the effect of any difference is to insert a spurious noise into one signal that is not seen at the other. This coherence condition of unity is not a necessary one, however. All that are necessary are the conditions leading to Eq. (5), in the vicinity of ω_e. The reason the coherence condition is introduced is that there is commercially available equipment that will readily compute γ^2.

Finally, Eq. (6) is not useful as Eq. (7) because it requires a large value of frequency. Since both signals will be falling off

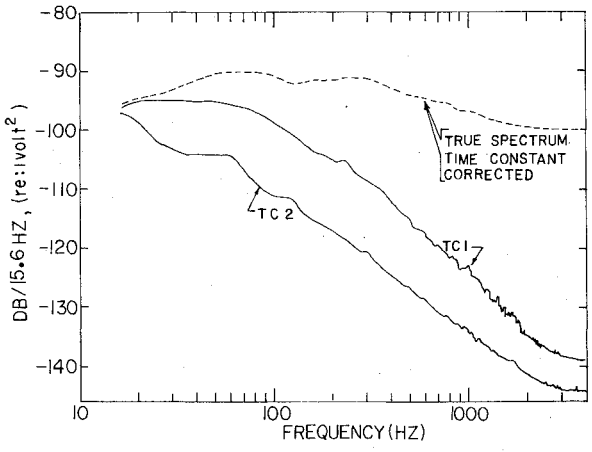

Fig. 1 Thermocouple spectra for two different thermocouples placed on the centerline at the exhaust plane of a gas turbine combustor.

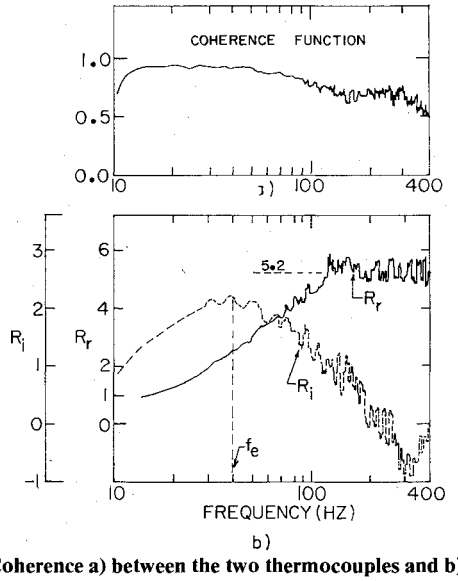

Fig. 2 Coherence a) between the two thermocouples and b) time constant determination for the thermocouples.

markedly at high frequency there is much more danger of dropping into the background noise at high frequency as compared with ω_e.

Experiment

To illustrate the procedure two commercial Chromel-Alumel thermocouples of wire diameters 0.001 in. (TC1) and 0.003 in. (TC2) were mounted less than 1/8 in. apart in the exhaust of a gas turbine combustor. Fixed bandwidth spectra (15.6 Hz) of the AC component of temperature are shown in Fig. 1. Also shown in Fig. 1 is the true temperature spectrum ($\eta^2 X_1 X_1^*$) after correction, knowing the thermocouple time constant. Since the time constant goes roughly as $D^{1.5}$, where D is the wire diameter, TC2 has a poorer response, which is also shown in Fig. 1. In this example the ratio of the time constant, τ_2/τ_1, should be roughly $3^{1.5} = 5.2$.

The time constant for TC1 was expected to lie between 1 and 10 msec, so finer bandwidth filtering (3.1 Hz), and a narrower frequency range (0-400 Hz) was chosen for time constant analysis in Fig. 2. In Fig. 2a, the coherence function is seen to be adequate within the band 10-400 Hz. In Fig. 2b are shown R_r and R_i. R_r shows the expected behavior of monotonically going from unity to τ_2/τ_1 as ω goes from zero to values much greater than $1/\tau_1$. R_i shows the expected behavior of a maximum at 40 *Hz*, corresponding to $\tau_1 = 1/(2\pi 40) = 4$ msec.

Some problems with this method are a) the AC component of temperature is usually substantially lower than the DC component so that high systems gains are needed and higher than desired background noise is usually evident, especially for the larger thermocouple, and b) the maximum is somewhat broad in R_i so that, in this example, about 25% error in τ may be expected. Problem a) is minimized by using thermocouples of nearly equivalent (but not equal) time constants. However, it appears that problem b) must be accepted as a limitation of the method.

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Preheated, Combustion-Driven Gasdynamic Lasers

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Introduction

EARLY in the development of the carbon dioxide gasdynamic laser (GDL), the variation in available laser energy as a function of laser gas stagnation temperature was readily noticeable. Experiments performed on a variety of laboratory devices[1,2] suggested that the laser energy available in a given device would be improved by operating with stagnation temperatures in the region of 2100 K, that is, at temperatures considerably above those of then-current devices. These experiments were generally performed on shock tube or arc-driven GDLs and, therefore, the gas composition was independent of any combustion process. For many applications, however, shock tube or arc-driven devices are not adequate. A solution is to use a combustion-driven device (first demonstrated by workers at AVCO Everett Research Laboratory[3]) in which reactants are burned to produce the necessary high-temperature laser gas. The disadvantage to this solution is that laser gas composition and stagnation temperature are no longer independent. In obtaining the higher temperatures necessary for more efficient operation, one encounters either undesirable laser gas compositions (which tend to negate the effect of the increased temperature) or fuel-oxidizer combinations that are not very practicable.

One method of eliminating the dependence of laser gas composition and temperature while still maintaining a

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